Information factor and measurement chain: when does a reflected or mixed signal carry state information?

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20 October 2026

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Status: Preprint draft, not peer reviewed. Text and calculations produced with AI assistance, the author being responsible.

Abstract

A signal carries information about a state θ only if different states produce distinguishable measurement distributions. This paper arranges known results from detection and estimation theory for reflection, scattering and mixing signals under one guiding question. The information factor $K_\theta = F\,\Delta\theta^2 = d'^2$ (Fisher information F, detectability d') is the signal-to-noise ratio of the difference signal; for two equiprobable states and Gaussian noise, $P_{\text{err}} = Q(d'/2)$. $K \approx 1$ corresponds to a 31 % error rate and is only a necessary threshold; robust decisions need $K \gtrsim 10$–16. A multiplicative split into source, material, coherence and geometry holds only under stated assumptions. Seven short numerical checks support the individual statements. Novelty is not claimed unless backed by a literature search; the contribution is organisation and reproducibility.

1 Guiding question

Which state information is distinguishable in the signal under the given measurement conditions? The answer is traced along the chain $X \to H_{\text{field}} \to H_{\text{mat}} \to H_{\text{mix}} \to H_{\text{meas}} \to Q \to Y$. If $p(Y \mid \theta_1) = p(Y \mid \theta_2)$ for $\theta_1 \neq \theta_2$, the state cannot be determined uniquely from Y; quantisation cannot increase mutual information (data-processing inequality).

2 Information factor

For $Y = s(\theta) + N$, $N \sim \mathcal{N}(0, \sigma^2 I)$: $F = \|\partial_\theta s\|^2/\sigma^2$ and $\mathrm{Var}(\hat{\theta}) \geq 1/F$. To distinguish θ from $\theta + \Delta\theta$: $d'^2 = \|\Delta s\|^2/\sigma^2 \approx F\Delta\theta^2 =: K_\theta$. For complex noise with total variance σ^2, F doubles; the convention must be stated. Total information obeys $I \leq \sum_i \frac{1}{2}\log_2(1 + \mathrm{SNR}_i)$ (real), i.e. about $\frac{N_{\text{DOF}}}{2}\log_2(1 + \mathrm{SNR})$.

Under (A1) signal $= A\sqrt{D_{\text{mat}}}\,V\,h(\theta)$ (material a power quantity, visibility an amplitude quantity), (A2) additive white Gaussian noise, (A3) independent factors, one obtains $K_\theta =$

$\mathrm{SNR}_0\, D_{\mathrm{mat}}\, V^2\, \|\partial_\theta h\|^2 \Delta\theta^2$. With shot noise, saturation, coupled coherence or nuisance parameters, the product form must be replaced.

3 Numerical evidence

Statement	Result
$P_{\mathrm{err}} = Q(d'/2)$	Monte Carlo 0.309 / 0.158 / 0.0227 / 0.00133 for $K = 1/4/16/36$
Signed direction factor	$S_e = +1,\ 0.69,\ 0,\ -1$; standing wave 0
Local cancellation	$\lvert\psi\rvert = 10^{-16}$ at the node, gradient maximal
Mixing products	linear field: 2 lines; nonlinear: lines at $f_2 - f_1$, $2f_1 - f_2$, ...
Aliasing	20 and 80 Hz at $f_s = 100$ Hz: difference $3 \cdot 10^{-14}$
Scattering vs transfer matrix	error 10^{-13} vs up to 10^9 (frustrated total reflection, one case)
Resolution	σ_{CRB}/a: 7.28 → 0.168 → 0.0020 for $q_{\max} a = 0.2/1/12$

The checks are consistency tests on single models, not experiments.

4 Delimitation

All building blocks are established (Shannon; Cover & Thomas; Cramér, Rao; Green & Swets; Barrett & Myers; Braunstein & Caves; Tsang, Nair & Lu). At most the common arrangement is original. That no similar formulation exists is **not established**; a systematic search is outstanding. Limits: Gaussian noise, Born approximation, scalar fields, no experiments. Falsification criteria and test programmes are in the book (ch. 11).

References

See the book, chapter “References”; entries marked † are not individually verified.

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